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Third Semester B.E. Degree Examination, Feb./Mar.2022 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1** a. Find a Fourier Series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence prove that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \dots$ (08 Marks)
- b. Obtain a Fourier series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$. (06 Marks)
- c. Find the half-range Fourier sine series of $f(x) = e^x$ in $0 < x < 1$. (06 Marks)

OR

- 2** a. Find the Fourier series expansion upto second harmonic using the following table of values:
- | | | | | | | | |
|---|-----|-----------------|------------------|-------|------------------|------------------|--------|
| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | π | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | 2π |
| y | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |
- (08 Marks)
- b. Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. (06 Marks)
- c. Obtain the Half range cosine series of $f(x) = x^2$ in $0 \leq x \leq \pi$. (06 Marks)

Module-2

- 3** a. Find the Fourier transform of the function, $f(x) = \begin{cases} 1, & \text{for } |x| \leq a \\ 0, & \text{for } |x| > a \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin ax}{x} dx$. (08 Marks)
- b. Find the Fourier cosine transform of $f(x) = e^{-ax}$, $a > 0$ (06 Marks)
- c. Solve $u_n + 3u_{n-1} - 4u_{n-2} = 0$ for $n \geq 2$ given $u_0 = 3$, $u_1 = -2$ using z-transform. (06 Marks)

OR

- 4** a. Find the Fourier sine transform of e^{-ax} , $a > 0$, $x > 0$ show that $\int_0^{\infty} \frac{x \sin mx}{a^2 + x^2} dx = \frac{\pi}{2} e^{-am}$, $m > 0$. (08 Marks)
- b. Find the z-transform of $\cosh\left(\frac{n\pi}{2} + \theta\right)$. (06 Marks)
- c. Find the inverse z-transform of, $\frac{3z^2 + z}{(5z - 1)(5z + 2)}$. (06 Marks)

**Module-3**

- 5 a. Find the correlation coefficient using the following table as values: (08 Marks)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

- b. Obtain an equation of the form $y = ax + b$ given that, (06 Marks)

x	0	5	10	15	20	25
y	12	15	17	22	24	30

- c. Apply Regula-Falsi method to find the root of $xe^x = \cos x$ in four approximations with four decimals in (0, 1). (06 Marks)

OR

- 6 a. Obtain the regression line of y on x for the following table of values: (08 Marks)

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

- b. Fit a parabola $y = a + bx + cx^2$ to the following data: (06 Marks)

x	20	40	60	80	100	120
y	5.5	9.1	14.9	22.8	33.3	46

- c. Find the root of the equation $x^4 - x - 9 = 0$ by Newton-Raphson method in three approximations with three decimal places. (Take $x_0 = 2$) (06 Marks)

Module-4

- 7 a. Use Newton's forward interpolation formula to find $y(8)$ from the table of values, (08 Marks)

x	0	5	10	15	20	25
y(x)	7	11	14	18	24	32

- b. Determine y at $x = 1$ using Newton's general interpolation formula given that, (06 Marks)

x	-4	-1	0	2	5
y(x)	1245	33	5	9	1335

- c. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Weddle's rule with $h = 1$. (06 Marks)

OR

- 8 a. Find $f(4)$ using Newton's Backward interpolation formula given that, (08 Marks)

x	0	1	2	3
y = f(x)	1	2	1	10

- b. Apply Lagrange's interpolation formula to find $y(x = 10)$ given that, (06 Marks)

x	5	6	9	11
y(x)	12	13	14	16

- c. Apply Simpson's $\frac{1}{3}$ formula to evaluate $\int_0^{120} V(t)dt$ given that, (06 Marks)

t	0	12	24	36	48	60	72	84	96	108	120
V(t)	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.5	5.4	9.0

(06 Marks)

**Module-5**

- 9 a. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$. (08 Marks)
- b. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's theorem with $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at, (0, 0, 0), (1, 0, 0) and (1, 1, 0). (06 Marks)
- c. Show that the geodesics on a plane are straight lines. (06 Marks)

OR

- 10 a. Find $\iint_S \vec{F} \cdot d\vec{S}$, where $F = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$ and S is the surface of the sphere having center at (3, -1, 2) and radius 3. (Use Gauss divergence theorem). (08 Marks)
- b. Derive Euler's equation with usual notations as, $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (06 Marks)
- c. Find the extremals of the functional,
$$\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3} \right) dx.$$
 (06 Marks)
